NCERT EXERCISE 4.1

1. Check whether the following are quadratic equations:

(i) $(x + 1)^2 = 2(x - 3)$

(ii) $x^2 - 2x = (-2)(3 - x)$

(iii) $(x - 2)(x + 1) = (x - 1)(x + 3)$

(iv) $(x - 3)(2x + 1) = x(x + 5)$

(v) $(2x - 1)(x - 3) = (x + 5)(x - 1)$

(vi) $x^2 + 3x + 1 = (x - 2)^2$

(vii) $(x + 2)^3 = 2x(x^2 - 1)$

(viii) $x^3 - 4x^2 - x + 1 = (x - 2)^3$

Sol. (i) $(x + 1)^2 = 2(x - 3)$

$\Rightarrow \quad x^2 + 2x + 1 = 2x - 6$

$\Rightarrow \quad x^2 + 2x - 2x + 1 + 6 = 0$

$\Rightarrow \quad x^2 + 7 = 0$

or $$x^2 + 0.x + 7 = 0$$

Which is of the form $ax^2 + bx + c = 0$.

Hence, the given equation is a quadratic equation.

(*ii*) $x^2 - 2x = (-2)(3 - x)$

$$\Rightarrow \quad x^2 - 2x = -6 + 2x$$

$$\Rightarrow \quad x^2 - 4x + 6 = 0$$

Which is of the form $ax^2 + bx + c = 0$. Hence, the given equation is a quadratic equation.

(*iii*) $(x - 2)(x + 1) = (x - 1)(x + 3)$

$$\Rightarrow \quad x^2 + x - 2x - 2 = x^2 + 3x - x - 3$$

$$\Rightarrow \quad x^2 + x - 2x - 2 - x^2 - 3x + x + 3 = 0$$

$$\Rightarrow \quad -3x + 1 = 0$$

$$\Rightarrow \quad 3x - 1 = 0$$

or $$0.x^2 + 3x + (-1) = 0$$

Which is of the form $ax^2 + bx + c = 0$, But $a = 0$. Hence, given equation is not a quadratic equation.

(*iv*) $(x - 3)(2x + 1) = x(x + 5)$

$\Rightarrow \quad 2x^2 + x - 6x - 3 = x^2 + 5x$

$\Rightarrow \quad 2x^2 + x - 6x - 3 - x^2 - 5x = 0$

$\Rightarrow \quad x^2 - 10x - 3 = 0$

Which is of the form $ax^2 + bx + c = 0$. Hence, the given equation is a quadratic equation.

(*v*) $(2x - 1)(x - 3) = (x + 5)(x - 1)$

$\Rightarrow \quad 2x^2 - 6x - x + 3 = x^2 - x + 5x - 5$

$\Rightarrow \quad 2x^2 - 6x - x + 3 - x^2 + x - 5x + 5 = 0$

$\Rightarrow \quad x^2 - 11x + 8 = 0$

Which is of the form $ax^2 + bx + c = 0$. Hence, the given equation is a quadratic equation.

(*vi*) $x^2 + 3x + 1 = (x - 2)^2$

$\Rightarrow \quad x^2 + 3x + 1 = x^2 + 4 - 4x$

$\Rightarrow \quad x^2 + 3x + 1 - x^2 - 4 + 4x = 0$

$\Rightarrow \quad 0.x^2 + 7x - 3 = 0$

Which is of the form $ax^2 + bx + c = 0$, but $a = 0$. Hence, the given equation is not a quadratic equation.

(*vii*) $(x + 2)^3 = 2x(x^2 - 1)$

$\Rightarrow \quad x^3 + 8 + 3.x.2(x + 2) = 2x^3 - 2x$

$\Rightarrow \quad x^3 + 8 + 6x^2 + 12x = 2x^3 - 2x$

$\Rightarrow \quad x^3 - 6x^2 - 14x - 8 = 0$

Which is not of the form $ax^2 + bx + c = 0$. Hence, the given equation is not a quadratic equation.

(*viii*) $x^3 - 4x^2 - x + 1 = (x - 2)^3$

$\Rightarrow \quad x^3 - 4x^2 - x + 1 = x^3 - 8 + 3.x(-2)(x - 2)$

$\Rightarrow \quad x^3 - 4x^2 - x + 1 = x^3 - 6x^2 + 12x - 8$

$\Rightarrow \quad 2x^2 - 13x + 9 = 0$

Which is of the form $ax^2 + bx + c = 0$. Hence, the given equation is a quadratic equation.

2. Represent the following situations in the form of quadratic equations:

(*i*) The area of a rectangular plot is 528 m^2. The length of the plot (in metres) is one more than twice its breadth. We need to find the length and breadth of the plot.

(*ii*) The product of two consecutive positive integers is 306. We need to find the integers.

(*iii*) Rohan's mother is 26 years older than him. The product of their ages (in years) 3 years

from now will be 360. We would like to find Rohan's present age.

(iv) A train travels a distance of 480 km at a uniform speed. If the speed had been 8 km/h less, then it would have taken 3 hours more to cover the same distance. We need to find the speed of the train.

1. (i) Let breadth of the rectangular plot be x m

$\Rightarrow$ Length of the plot = $(2x + 1)$m

Area of a rectangular plot = $l \times b$

$\Rightarrow \quad 528 = (2x + 1)\,x$

$\Rightarrow \quad 528 = 2x^2 + x$

$\Rightarrow \quad 2x^2 + x - 528 = 0$

Which is the required quadratic equation.

(ii) Let two consecutive integers be x and $x + 1$.

Then, $\quad x\,(x + 1) = 306$

$\Rightarrow \quad x^2 + x - 306 = 0$

Which is the required quadratic equation.

(iii) Let present age of Rohan be x years

Rohan's mother's present age be $(x + 26)$ years

After 3 years, Rohan's age = $(x + 3)$ years

After 3 years, Rohan's mother's age

$= (x + 26 + 3)$ years

ATQ $(x + 3)\,(x + 29) = 360$

$\Rightarrow \quad x^2 + 32x - 273 = 0$

Which is the required quadratic equation.

(iv) Let speed of the train be x km/h

Total distance to be covered = 480 km

Time= $\dfrac{\text{distance}}{\text{speed}} = \dfrac{480}{x}$ hours

Decreased speed of the train = $(x - 8)$ km/h

Now, $\quad \text{time} = \dfrac{480}{x-8}$ hours

ATQ $\quad \dfrac{480}{x-8} - \dfrac{480}{x} = 3$

$\Rightarrow 480\left[\dfrac{1}{x-8} - \dfrac{1}{x}\right] = 3$

$\Rightarrow 480\left[\dfrac{x - x + 8}{x(x-8)}\right] = 3$

$\Rightarrow \quad 480 \times 8 = 3x\,(x - 8)$

$\Rightarrow \quad 3840 = 3x^2 - 24x$

$\Rightarrow \quad 3x^2 - 24x - 3840 = 0$

or $\quad x^2 - 8x - 1280 = 0$

Which is the required quadratic equation.

NCERT EXERCISE 4.2

1. Find the roots of the following quadratic equations by factorisation:

(i) $x^2 - 3x - 10 = 0$

(ii) $2x^2 + x - 6 = 0$

(iii) $\sqrt{2}x^2 + 7x + 5\sqrt{2} = 0$ [AI 2017(C)]

(iv) $2x^2 - x + \frac{1}{8} = 0$

(v) $100x^2 - 20x + 1 = 0$

Sol. (i) $x^2 - 3x - 10 = 0 \Rightarrow x^2 - 5x + 2x - 10 = 0$

$\Rightarrow x(x-5) + 2(x-5) = 0$

$\Rightarrow (x+2)(x-5) = 0$

$\Rightarrow x + 2 = 0$ or $x - 5 = 0$

$\Rightarrow x = -2$ or $x = 5$

Hence, the roots are 5, – 2.

(ii) $2x^2 + x - 6 = 0$

$\Rightarrow 2x^2 + 4x - 3x - 6 = 0$

$\Rightarrow 2x(x+2) - 3(x+2) = 0$

$\Rightarrow (2x-3)(x+2) = 0$

$\Rightarrow 2x - 3 = 0$ or $x + 2 = 0$

$\Rightarrow 2x = 3$ or $x = -2$

$\Rightarrow x = \frac{3}{2}$ or $x = -2$

Hence, the roots are $-2, \frac{3}{2}$

(iii) $\sqrt{2}x^2 + 7x + 5\sqrt{2} = 0$

$\Rightarrow \sqrt{2}x^2 + 5x + 2x + 5\sqrt{2} = 0$

$\Rightarrow x(\sqrt{2}x + 5) + \sqrt{2}(\sqrt{2}x + 5) = 0$

$\Rightarrow (x + \sqrt{2})(\sqrt{2}x + 5) = 0$

$\Rightarrow x + \sqrt{2} = 0$ or $\sqrt{2}x + 5 = 0$

$\Rightarrow x = -\sqrt{2}$ or $x = -\frac{5}{\sqrt{2}}$

Hence, the roots are $-\sqrt{2}, -\frac{5}{\sqrt{2}}$.

$$x = \frac{-b+\sqrt{D}}{2a}, \frac{-b-\sqrt{D}}{2a}$$

(*ii*) If D = 0, Solution/roots of the quadratic equation are given by $x = \frac{-b}{2a}$.

(*iii*) If D < 0, equation has no real roots.

(*iv*) $2x^2 - x + \frac{1}{8} = 0$

$\Rightarrow \frac{16x^2 - 8x + 1}{8} = 0$

$\Rightarrow 16x^2 - 4x - 4x + 1 = 0$

$\Rightarrow 4x(4x - 1) - 1(4x - 1) = 0$

$\Rightarrow (4x - 1)(4x - 1) = 0$

$\Rightarrow 4x - 1 = 0$

or $4x - 1 = 0$

$\Rightarrow x = \frac{1}{4}$

or $x = \frac{1}{4}$

Hence, the roots are $\frac{1}{4}, \frac{1}{4}$.

(*v*) $100x^2 - 20x + 1 = 0$

$\Rightarrow 100x^2 - 10x - 10x + 1 = 0$

$\Rightarrow 10x(10x - 1) - 1(10x - 1) = 0$

$\Rightarrow (10x - 1)(10x - 1) = 0$

$\Rightarrow 10x - 1 = 0$ or $10x - 1 = 0$

$\Rightarrow x = \frac{1}{10}$ or $x = \frac{1}{10}$

Hence, the roots are $\frac{1}{10}, \frac{1}{10}$.

2. Solve the following situations mathematically:

(*i*) John and Jivanti together have 45 marbles. Both of them lost 5 marbles each and the product of the number of marbles they now have is 124. We would like to find out how many marbles they had to start with.

(*ii*) A cottage industry produces a certain number of toys in a day. The cost of production of each toy (in rupees) was found to be 55 minus the number of toys produced in a day. On a

particular day, the total cost of production was ₹ 750. We would like to find out the number of toys produced on that day.

Sol. (*i*) Let the number of marbles John had be x

Then the number of marbles Jivanti had be $45 - x$

The number of marbles left with John, when he lost 5 marbles $= x - 5$

The number of marbles left with Jivanti, when she lost 5 marbles $= 45 - x - 5 = 40 - x$

ATQ $(x - 5)(40 - x) = 124$

$\Rightarrow 40x - x^2 - 200 + 5x = 124$

$\Rightarrow x^2 - 45x + 324 = 0$

$\Rightarrow x^2 - 36x - 9x + 324 = 0$

$\Rightarrow x(x - 36) - 9(x - 36) = 0$

$\Rightarrow (x - 9)(x - 36) = 0$

$\Rightarrow x - 9 = 0$ or $x - 36 = 0$

$\Rightarrow x = 9$ or $x = 36$

if $x = 9$, then $45 - x = 45 - 9 = 36$

if $x = 36$, then $45 - x = 45 - 36 = 9$

Hence, number of marbles they had to start with 9 and 36.

(*ii*) Let the number of toys produced in a day be x

Then, cost of production of each toy on that day be ₹ $(55-x)$

Total cost of production on that day = ₹ $x(55-x)$

ATQ $x(55 - x) = 750$

$\Rightarrow 55x - x^2 = 750$

$\Rightarrow x^2 - 55x + 750 = 0$

$\Rightarrow x^2 - 25x - 30x + 750 = 0$

$\Rightarrow (x - 30)(x - 25) = 0$

$\Rightarrow x - 30 = 0$ or $x - 25 = 0$

$\Rightarrow x = 30$ or $x = 25$

Number of toys produced on that day was 25 or 30.

3. Find two numbers whose sum is 27 and product is 182.

Sol. Let one number be x, then other number be $27 - x$

ATQ $x(27 - x) = 182$

$\Rightarrow x^2 - 27x + 182 = 0$

$\Rightarrow x^2 - 14x - 13x + 182 = 0$

$\Rightarrow x(x - 14) - 13(x - 14) = 0$

$$\Rightarrow \quad (x-13)(x-14) = 0$$

$$\Rightarrow \quad x-13 = 0 \text{ or } x-14=0$$

$$\Rightarrow \quad x = 13 \text{ or } x = 14$$

Hence, the numbers are 13 and 14.

4. Find two consecutive positive integers, sum of whose squares is 365.

Sol. Let the two consecutive integers be x and $x + 1$

ATQ $\quad x^2 + (x+1)^2 = 365$

$$\Rightarrow \quad x^2 + x^2 + 2x + 1 = 365$$

$$\Rightarrow \quad 2x^2 + 2x - 364 = 0$$

$$\Rightarrow \quad x^2 + x - 182 = 0$$

$$\Rightarrow \quad x^2 + 14x - 13x - 182 = 0$$

$$\Rightarrow \quad x(x+14) - 13(x+14) = 0$$

$$\Rightarrow \quad (x-13)(x+14) = 0$$

$\Rightarrow \quad x = 13, -14 \quad$ (−14 is rejected because it is a negative integer)

Hence, the two consecutive positive integers are 13 and 13 + 1 = 14.

5. The altitude of a right triangle is 7 cm less than its base. If the hypotenuse is 13 cm, find the other two sides.

Sol. Let base be x cm, then height be $(x - 7)$ cm

By Pythagoras Theorem,

$$(\text{base})^2 + (\text{height})^2 = (\text{hypotenuse})^2$$

$$\Rightarrow \quad x^2 + (x-7)^2 = (13)^2$$

$$\Rightarrow \quad x^2 + x^2 + 49 - 14x = 169$$

$$\Rightarrow \quad 2x^2 - 14x - 120 = 0$$

$$\Rightarrow \quad x^2 - 7x - 60 = 0$$

$$\Rightarrow \quad x^2 - 12x + 5x - 60 = 0$$

$$\Rightarrow \quad x(x-12) + 5(x-12) = 0$$

$$\Rightarrow \quad (x+5)(x-12) = 0$$

$$\Rightarrow \quad x - 12 = 0 \text{ or } x + 5 = 0$$

$$\Rightarrow \quad x = 12 \text{ or } x = -5$$

(− 5 is rejected as sides can never be negative)

$\Rightarrow$ Base = 12 cm and altitude = 12 − 7 = 5 cm

6. A cottage industry produces a certain number of pottery articles in a day. It was observed on a particular day that the cost of production of each article (in rupees) was 3 more than twice the number of articles produced on that day. If the total cost of production on that day was ₹ 90, find the number of articles produced and the cost of each article.

Sol. Let total number of pottery articles produced in a day be x

$$\text{Cost of production} = ₹\frac{90}{x}$$

ATQ $\quad 2x + 3 = \frac{90}{x}$

$\Rightarrow \quad x(2x+3) = 90$

$\Rightarrow \quad 2x^2 + 3x = 90$

$\Rightarrow \quad 2x^2 + 3x - 90 = 0$

NCERT EXERCISE 4.3

1. Find the roots of the following quadratic equations, if they exist, by the method of completing the square:

(i) $2x^2 - 7x + 3 = 0$

(ii) $2x^2 + x - 4 = 0$

(iii) $4x^2 + 4\sqrt{3}x + 3 = 0$

(iv) $2x^2 + x + 4 = 0$

Sol. (i) $\quad 2x^2 - 7x + 3 = 0$

$\Rightarrow \quad 2x^2 - 7x = -3$

Dividing both sides by 2, we get

$$x^2 - \frac{7}{2}x = \frac{-3}{2}$$

Adding the square of ($\frac{1}{2}$coefficient of x) on both sides, we get

$$x^2 - \frac{7}{2}x + \left(\frac{-7}{4}\right)^2 = \frac{-3}{2} + \left(\frac{-7}{4}\right)^2$$

$\Rightarrow \quad \left(x - \frac{7}{4}\right)^2 = -\frac{3}{2} + \frac{49}{16}$

$\Rightarrow \quad \left(x - \frac{7}{4}\right)^2 = \frac{-24 + 49}{16}$

$\Rightarrow \quad \left(x - \frac{7}{4}\right)^2 = \frac{25}{16}$

$\Rightarrow \quad \left(x - \frac{7}{4}\right)^2 = \left(\frac{5}{4}\right)^2$

$\Rightarrow \quad x - \frac{7}{4} = \pm\frac{5}{4}$

$\Rightarrow \quad x - \frac{7}{4} = \frac{5}{4}$ and $x - \frac{7}{4} = -\frac{5}{4}$

$\Rightarrow \quad x = \frac{5}{4} + \frac{7}{4}$

and $\quad x = -\frac{5}{4} + \frac{7}{4}$

$\Rightarrow \quad x = \frac{12}{4}$ and $x = \frac{2}{4}$

$\Rightarrow \quad x = 3$ and $x = \frac{1}{2}$

Hence, $x = 3, \frac{1}{2}$ are the roots of given quadratic equation.

$\Rightarrow \quad 2x^2 + 15x - 12x - 90 = 0$

$\Rightarrow \quad x(2x + 15) - 6(2x + 15) = 0$

$\Rightarrow \quad (2x + 15)(x - 6) = 0$

$\Rightarrow \quad 2x = -15 \text{ or } x - 6 = 0$

$\Rightarrow \quad x = -\frac{15}{2}$ ($-\frac{15}{2}$ is rejected) or $x = 6$

$\therefore$ Number of articles produced per day = 6

Cost of production per article = $\frac{90}{6}$ = ₹ 15

(*ii*) $2x^2 + x - 4 = 0$

$\Rightarrow \quad 2x^2 + x = 4$

On dividing both sides by 2, we get.

$$x^2 + \frac{1}{2}x = 2$$

Adding the square of ($\frac{1}{2}$ coefficient of x) on both sides, we get

$$x^2 + \frac{1}{2}x + \left(\frac{1}{2} \times \frac{1}{2}\right)^2 = 2 + \left(\frac{1}{2} \times \frac{1}{2}\right)^2$$

$\Rightarrow \quad x^2 + \frac{1}{2}x + \frac{1}{16} = 2 + \frac{1}{16}$

$\Rightarrow \quad \left(x + \frac{1}{4}\right)^2 = \frac{33}{16}$

or $\quad \left(x + \frac{1}{4}\right)^2 = \left(\frac{\sqrt{33}}{4}\right)^2$

$\Rightarrow \quad x + \frac{1}{4} = \pm\frac{\sqrt{33}}{4}$

$\Rightarrow \quad x + \frac{1}{4} = \frac{\sqrt{33}}{4}$

and $\quad x + \frac{1}{4} = -\frac{\sqrt{33}}{4}$

$\Rightarrow \quad x = \frac{\sqrt{33}}{4} - \frac{1}{4}$

and $\quad x = -\frac{\sqrt{33}}{4} - \frac{1}{4}$

$\Rightarrow \quad x = \frac{\sqrt{33} - 1}{4}$

and $\quad x = \frac{-\sqrt{33} - 1}{4}$

Hence, the roots are $\frac{\sqrt{33} - 1}{4}$ and $\frac{-\sqrt{33} - 1}{4}$

(*iii*) $4x^2 + 4\sqrt{3}x + 3 = 0$

$\Rightarrow \quad x^2 + \frac{4}{4}\sqrt{3}x + \frac{3}{4} = 0$

$\Rightarrow \quad x^2 + \sqrt{3}x = -\frac{3}{4}$

Now adding square of ($\frac{1}{2}$ coefficient of x) on both sides, we get

$$\Rightarrow \quad x^2 + \sqrt{3}x + \left(\frac{\sqrt{3}}{2}\right)^2 = \frac{-3}{4} + \left(\frac{\sqrt{3}}{2}\right)^2$$

$$\Rightarrow \quad \left(x + \frac{\sqrt{3}}{2}\right)^2 = -\frac{3}{4} + \frac{3}{4}$$

$$\Rightarrow \quad \left(x + \frac{\sqrt{3}}{2}\right)^2 = 0$$

$$\Rightarrow \quad x + \frac{\sqrt{3}}{2} = 0$$

$$\Rightarrow \quad x = -\frac{\sqrt{3}}{2}$$

Hence, the roots are $\frac{-\sqrt{3}}{2}$ and $\frac{-\sqrt{3}}{2}$.

(*iv*) $$2x^2 + x + 4 = 0$$

$$\Rightarrow \quad x^2 + \frac{1}{2}x + \frac{4}{2} = 0$$

$$\Rightarrow \quad x^2 + \frac{1}{2}x = -2$$

Now adding square of ($\frac{1}{2}$ coefficient of x) on both sides, we get

$$\Rightarrow \quad x^2 + \frac{1}{2}x + \left(\frac{1}{4}\right)^2 = -2 + \left(\frac{1}{4}\right)^2$$

$$\Rightarrow \quad \left(x + \frac{1}{4}\right)^2 = -2 + \frac{1}{16}$$

$$\Rightarrow \quad \left(x + \frac{1}{4}\right)^2 = \frac{-32+1}{16}$$

$$\Rightarrow \quad \left(x + \frac{1}{4}\right)^2 = \frac{-31}{16}$$

$$\Rightarrow \quad \left(x + \frac{1}{4}\right)^2 = \left(\frac{\sqrt{-31}}{16}\right)^2$$

Which is impossible as square root of negative integers is not possible.

Hence, the roots do not exist.

2. Find the roots of the following quadratic equations by applying the quadratic formula:

(*i*) $2x^2 - 7x + 3 = 0$

(*ii*) $2x^2 + x - 4 = 0$

(*iii*) $4x^2 + 4\sqrt{3}x + 3 = 0$

(*iv*) $2x^2 + x + 4 = 0$

Sol. (*i*). $2x^2 - 7x + 3 = 0$

This is of the form $ax^2 + bx + c = 0$, where $a = 2$, $b = -7$ and $c = 3$

Discriminant (D) $= b^2 - 4ac$

$= (-7)^2 - 4 \times 2 \times 3 = 49 - 24 = 25 > 0$

Let roots be α and β

$$\alpha = \frac{-b + \sqrt{D}}{2a}$$

$$= \frac{-(-7) + \sqrt{25}}{2 \times 2}$$

$= \frac{7+5}{4} = \frac{12}{4} = 3$

$\beta = \frac{-b-\sqrt{D}}{2a}$

$= \frac{-(-7)-\sqrt{25}}{2\times 2} = \frac{7-5}{4} = \frac{2}{4} = \frac{1}{2}$

Hence, the roots are 3, $\frac{1}{2}$.

(*ii*) $2x^2 + x - 4 = 0$

This is of the form $ax^2 + bx + c = 0$,

where $a = 2$, $b = 1$ and $c = -4$

Discriminant (D) $= b^2 - 4ac = (1)^2 - 4 \times 2\,(-4)$

$= 1 + 32 = 33 > 0$

Let roots be α and β

$\alpha = \frac{-b+\sqrt{D}}{2a} = \frac{-1+\sqrt{33}}{4}$

$\beta = \frac{-b-\sqrt{D}}{2a} = \frac{-1-\sqrt{33}}{4}$

Hence, the roots are $\frac{-1+\sqrt{33}}{4}, \frac{-1-\sqrt{33}}{4}$.

(*iii*) $4x^2 + 4\sqrt{3}x + 3 = 0$

This is of the form $ax^2 + bx + c = 0$,

where $a = 4$, $b = 4\sqrt{3}$ and $c = 3$.

Discriminant (D) $= b^2 - 4ac$

$= (4\sqrt{3})^2 - 4 \times 4 \times 3 = 48 - 48 = 0$

Roots are

$\alpha = \frac{-b+\sqrt{D}}{2a}$

$= \frac{-4\sqrt{3}+0}{8} = \frac{-4\sqrt{3}}{8} = \frac{-\sqrt{3}}{2}$

and $\beta = \frac{-b-\sqrt{D}}{2a}$

$= \frac{-4\sqrt{3}-0}{8} = \frac{-4\sqrt{3}}{8} = \frac{-\sqrt{3}}{2}$

Hence, the roots are $\frac{-\sqrt{3}}{2}, \frac{-\sqrt{3}}{2}$.

(*iv*) $2x^2 + x + 4 = 0$

This is of the form $ax^2 + bx + c = 0$,

where $a = 2$, $b = 1$ and $c = 4$

Discriminant (D) $= b^2 - 4ac$

$= (1)^2 - 4 \times 2 \times 4 = 1 - 32 = -31 < 0$

Roots are not possible as discriminant is negative.

3. Find the roots of the following equations:

(*i*) $x - \frac{1}{x} = 3, x \neq 0$

(*ii*) $\frac{1}{x+4} - \frac{1}{x-7} = \frac{11}{30}, x \neq -4, 7$

Sol. (*i*) $\frac{x}{1} - \frac{1}{x} = 3$

$\Rightarrow \quad \frac{x^2 - 1}{x} = 3$

$\Rightarrow \quad x^2 - 1 = 3x$

or $\quad x^2 - 3x - 1 = 0$

This is of the form $ax^2 + bx + c = 0$,

where $a = 1$, $b = -3$ and $c = -1$

Discriminant (D) $= b^2 - 4ac = (-3)^2 - 4 \times 1 \times (-1) = 9 + 4 = 13$

Let roots be α and β;

$$\alpha = \frac{-b + \sqrt{D}}{2a}$$

$$= \frac{-(-3) + \sqrt{13}}{2 \times 1} = \frac{3 + \sqrt{13}}{2}$$

$$\beta = \frac{-b - \sqrt{D}}{2a}$$

$$= \frac{-(-3) - \sqrt{13}}{2 \times 1} = \frac{3 - \sqrt{13}}{2}$$

Hence, roots are $\frac{3 + \sqrt{13}}{2}, \frac{3 - \sqrt{13}}{2}$

(*ii*) $\quad \frac{1}{x+4} - \frac{1}{x-7} = \frac{11}{30}$

$\Rightarrow \quad \frac{x - 7 - (x + 4)}{(x + 4)(x - 7)} = \frac{11}{30}$

$\Rightarrow \quad \frac{x - 7 - x - 4}{x^2 - 7x + 4x - 28} = \frac{11}{30}$

$\Rightarrow \quad \frac{-11}{x^2 - 3x - 28} = \frac{11}{30}$

or $\quad \frac{-1}{x^2 - 3x - 28} = \frac{1}{30}$

$\Rightarrow \quad x^2 - 3x - 28 + 30 = 0$

$\Rightarrow \quad x^2 - 3x + 2 = 0$

This is of the form $ax^2 + bx + c = 0$,

where $a = 1$, $b = -3$ and $c = 2$

Discriminant (D) $= b^2 - 4ac$

$= (-3)^2 - 4 \times 1 \times 2 = 9 - 8 = 1$

Let roots be α and β,

$$\alpha = \frac{-b + \sqrt{D}}{2a}$$

$$= \frac{-(-3) + \sqrt{1}}{2 \times 1}$$

$$= \frac{3 + 1}{2} = \frac{4}{2} = 2$$

$$\beta = \frac{-b - \sqrt{D}}{2a}$$

$$= \frac{-(-3) - \sqrt{1}}{2 \times 1} = \frac{3 - 1}{2} = \frac{2}{2} = 1$$

Hence, roots are 2 and 1.

NCERT EXERCISE 4.4

1. **Find the nature of the roots of the following quadratic equations. If the real roots exist, find them:**

   (*i*) $2x^2 - 3x + 5 = 0$ (*ii*) $3x^2 - 4\sqrt{3}x + 4 = 0$

   (*iii*) $2x^2 - 6x + 3 = 0$

Sol. (*i*) $2x^2 - 3x + 5 = 0$

This is of the form $ax^2 + bx + c = 0$,

where $a = 2, b = -3$ and $c = 5$

Discriminant $(D) = b^2 - 4ac$

$= (-3)^2 - 4 \times 2 \times 5$

$= 9 - 40 = -31 < 0$

Hence, no real roots exist.

(*ii*) $3x^2 - 4\sqrt{3}x + 4 = 0$

This is of the form $ax^2 + bx + c = 0$,

where $a = 3, b = -4\sqrt{3}$ and $c = 4$

Discriminant $(D) = b^2 - 4ac$

$= (-4\sqrt{3})^2 - 4 \times 3 \times 4$

$= 48 - 48 = 0$

Since, $D = 0$

Hence, real and equal roots exist.

$$x = \frac{-b \pm \sqrt{D}}{2a}$$

$$\Rightarrow \quad x = \frac{-(-4\sqrt{3}) \pm 0}{2 \times 3} = \frac{4\sqrt{3}}{6}$$

$\therefore$ Roots are $\frac{2\sqrt{3}}{3}, \frac{2\sqrt{3}}{3}$.

(*iii*) $2x^2 - 6x + 3 = 0$

This is of the form $ax^2 + bx + c = 0$,

where $a = 2, b = -6$ and $c = 3$

Discriminant $(D) = b^2 - 4ac$

$= (-6)^2 - 4 \times 2 \times 3$

$= 36 - 24 = 12 > 0$

Hence, distinct real roots exist.

$$x = \frac{-b \pm \sqrt{D}}{2a} = \frac{-(-6) \pm \sqrt{12}}{2 \times 2}$$

(iii) If $a + b + c = 0$ then roots of quadratic equation are 1 and $\frac{c}{a}$.

(iv) If $a + c = b$ or $a - b + c = 0$ then roots of the quadratic equation are -1 and $-\frac{c}{a}$.

$$= \frac{6 \pm \sqrt{4 \times 3}}{2 \times 2} = \frac{6 \pm 2\sqrt{3}}{4}$$

$$= \frac{2(3 \pm \sqrt{3})}{4} \text{ or } \frac{3 \pm \sqrt{3}}{2}$$

Hence, roots are $\frac{3+\sqrt{3}}{2}, \frac{3-\sqrt{3}}{2}$

2. Find the values of k for each of the following quadratic equations, so that they have two equal roots.

(i) $2x^2 + kx + 3 = 0$

(ii) $kx(x - 2) + 6 = 0$

Sol. (i) $2x^2 + kx + 3 = 0$

This is of the form $ax^2 + bx + c = 0$,

where, $a = 2$, $b = k$ and $c = 3$

Discriminant (D) $= b^2 - 4ac$

$= k^2 - 4 \times 2 \times 3$

$= k^2 - 24$

For equal roots, $D = 0$

$\Rightarrow k^2 - 24 = 0$

$\Rightarrow k^2 = 24$ or $k = \pm\sqrt{24}$

$\Rightarrow k = \pm\sqrt{4 \times 6} = \pm 2\sqrt{6}$

(ii) $kx(x - 2) + 6 = 0$

$\Rightarrow kx^2 - 2kx + 6 = 0$

This is of the form $ax^2 + bx + c = 0$,

where $a = k$, $b = -2k$ and $c = 6$

Discriminant (D) $= b^2 - 4ac$

$= (-2k)^2 - 4 \times k \times 6$

$= 4k^2 - 24k$

For equal roots, $D = 0$

$\Rightarrow 4k^2 - 24k = 0$

$\Rightarrow k(4k - 24) = 0$

$\Rightarrow k = 0$ (not possible) or $4k - 24 = 0$

$\Rightarrow 4k = 24 \Rightarrow k = \frac{24}{4} = 6$